Symbolic Density Models of One-in-a-Billion Statistical Tails via Importance Sampling and Genetic Programming

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Genetic Programming Theory and Practice
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solido
DESIGN AUTOMATION

Tails

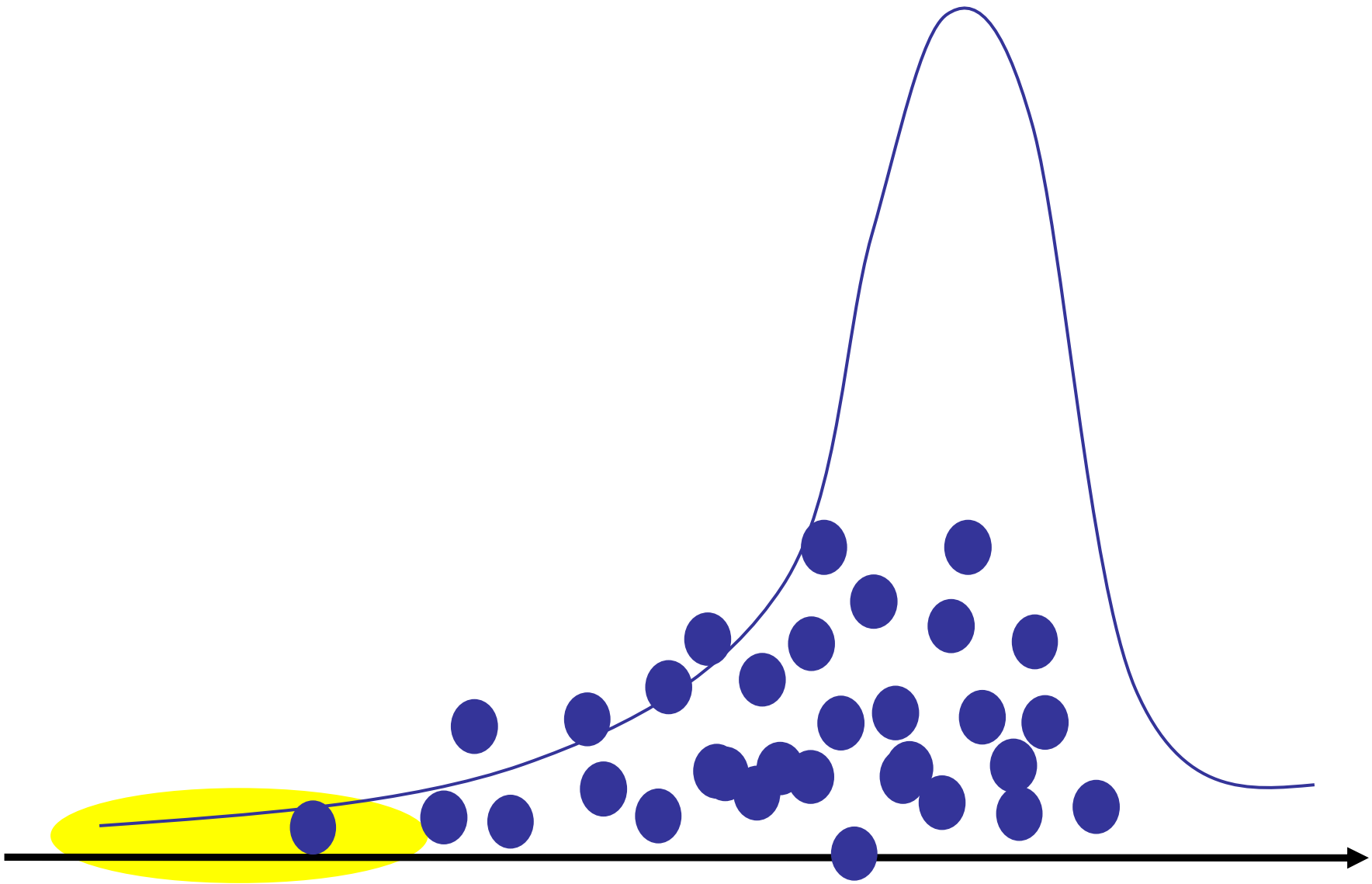
Tails

Tails

Tails

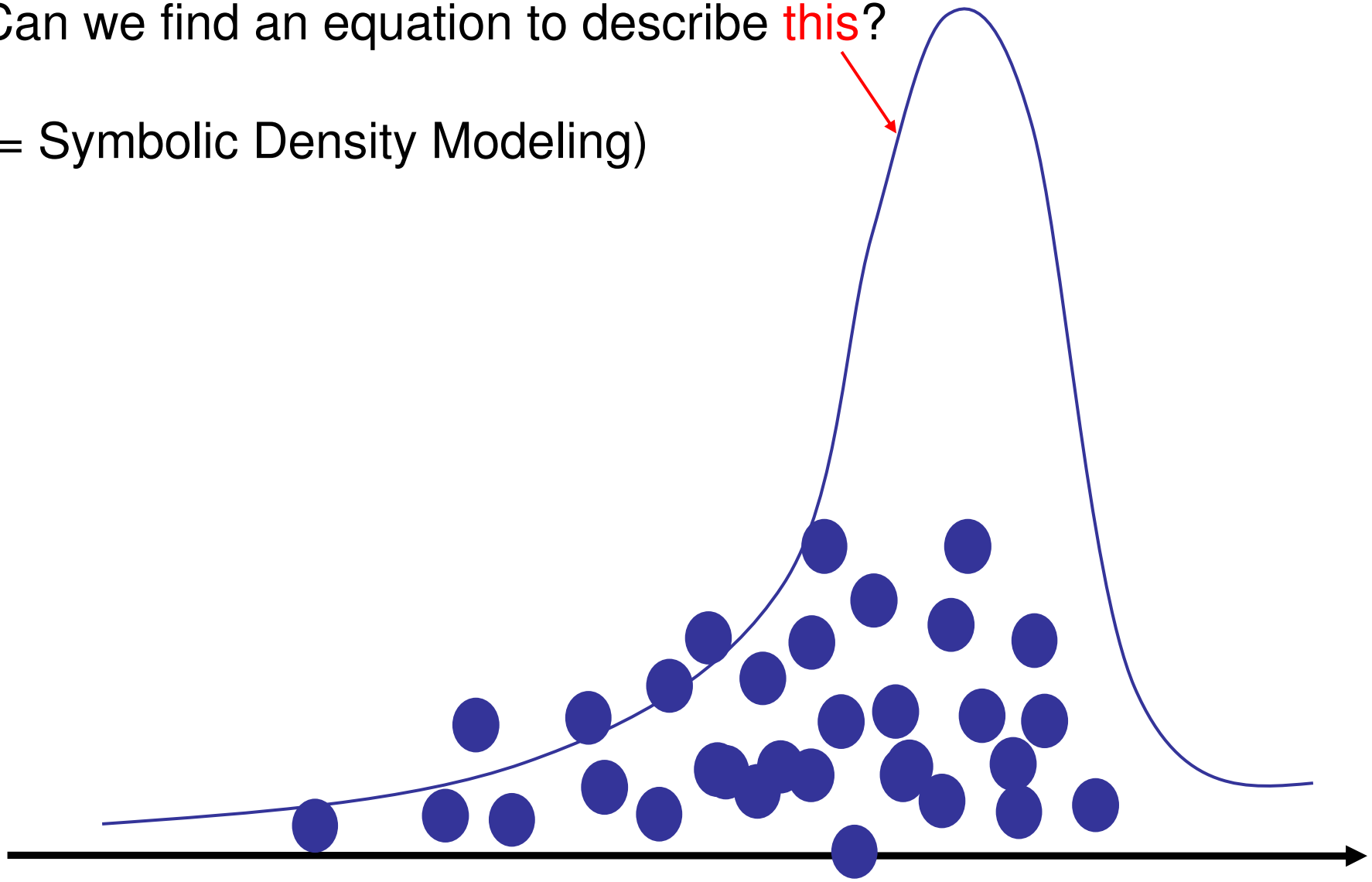
Tails

Tails



Can we find an equation to describe this?

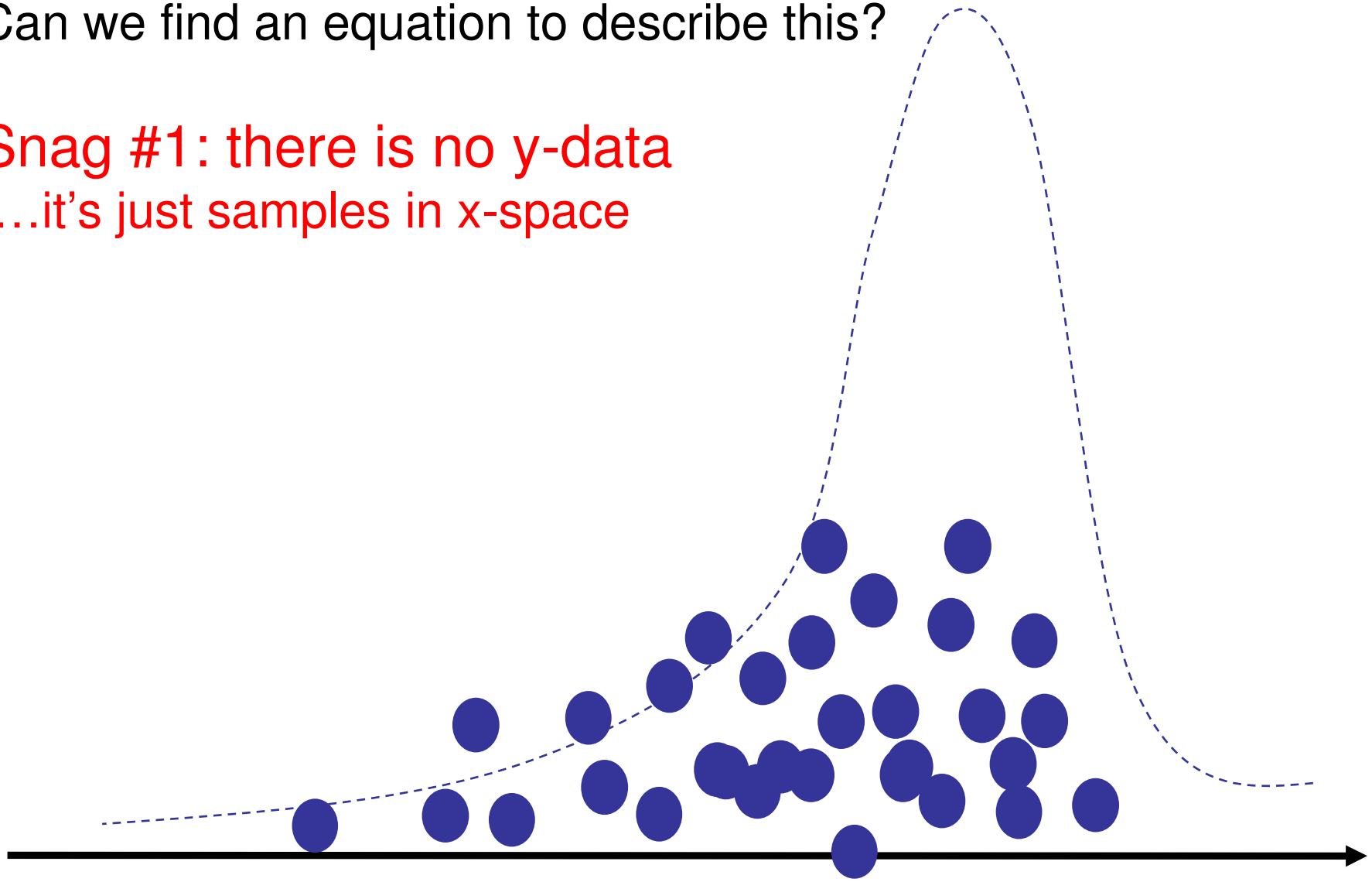
(= Symbolic Density Modeling)



Can we find an equation to describe this?

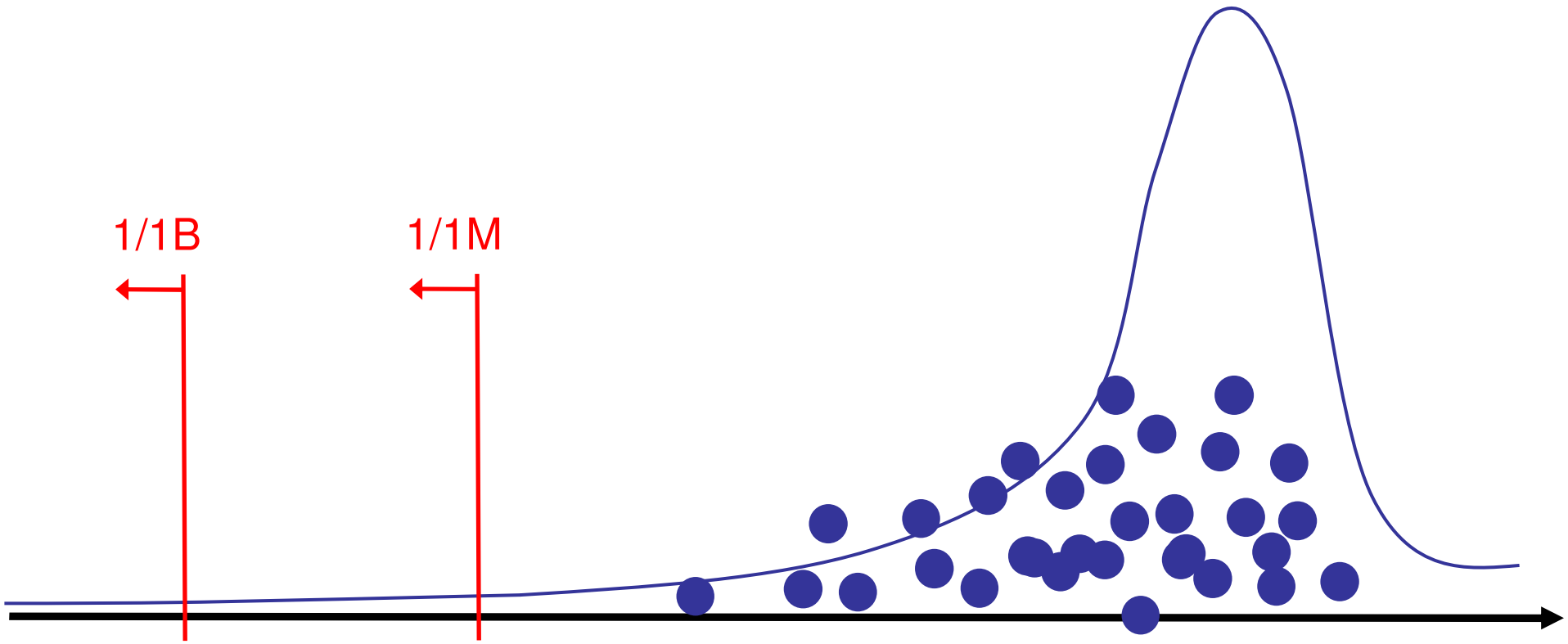
Snag #1: there is no y-data

…it's just samples in x-space



Snag #2: by definition, statistical tails are improbable

- can we generate 1B samples?
- can we model-fit 1B samples?



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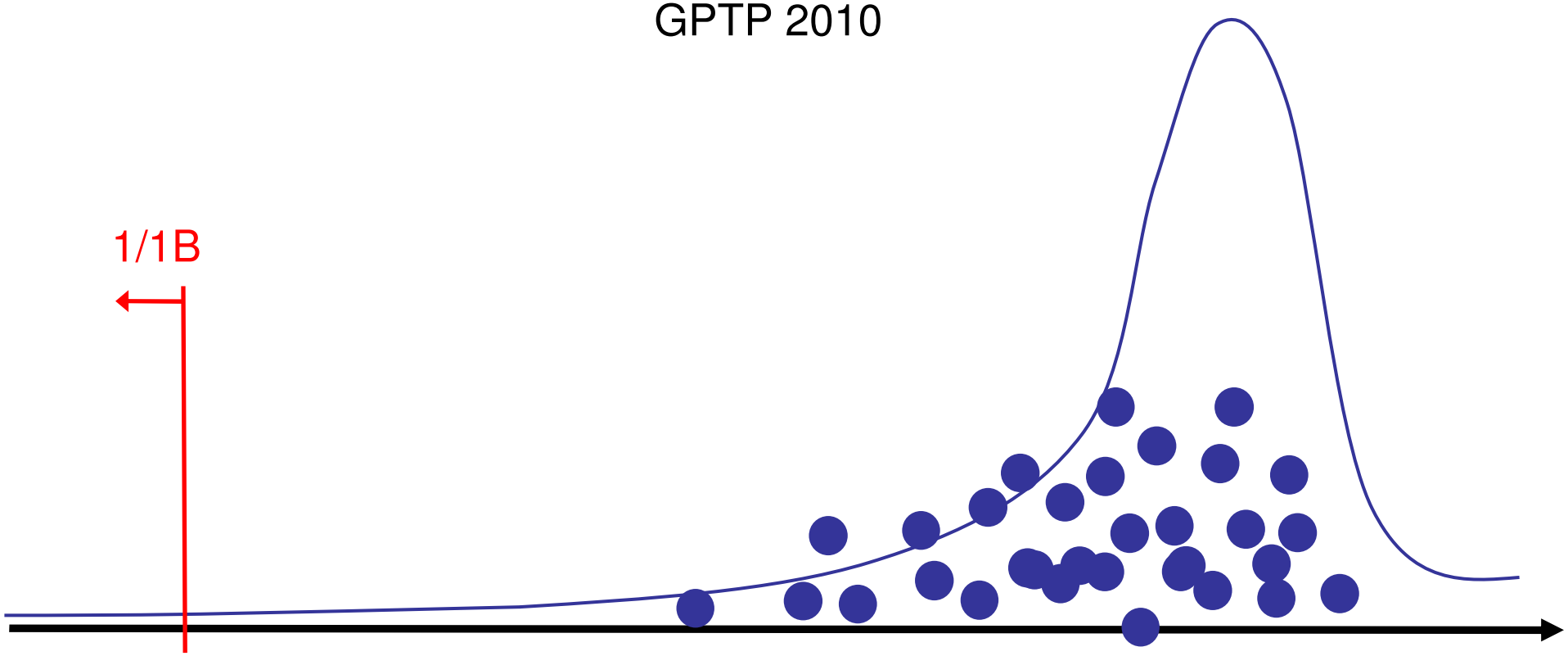
Of One-in-a-Billion Statistical Tails

Trent McConaghy
GPTP 2010



(Why) Does Anyone Care

About Statistical Tails?


iPod
Music
Videos
Photos
Podcasts
Extras
Settings
Shuffle Songs
Now Playing
MENU

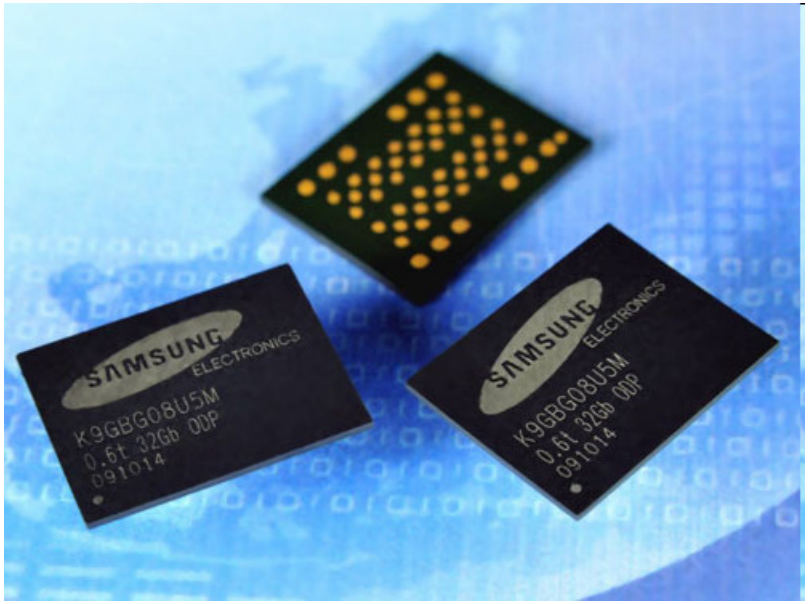

cingular
9:41 AM
Text
Calendar
Photos
Camera
Calculator
Stocks
Maps
Weather
Notes
Clock
Settings
Phone
Mail
Web
iPod


SONY
Cyber-shot
8.1
MEGA PIXELS

8GB memory

32GB memory

32GB memory (SD card)


SAMSUNG
ELECTRONICS
K9GBG08U5M
0.6t 32Gb 0DP
091014
SAMSUNG
ELECTRONICS
K9GBG08U5M
0.6t 32Gb 0DP
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iPod
Music
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Photos
Podcasts
Extras
Settings
Shuffle Songs
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MENU


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9:41 AM
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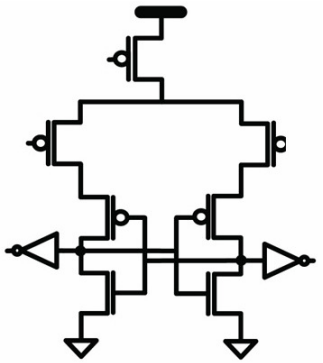
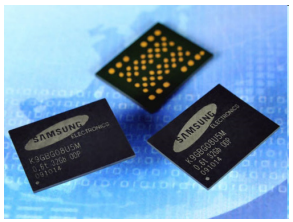
Weather
Notes
Clock
Settings

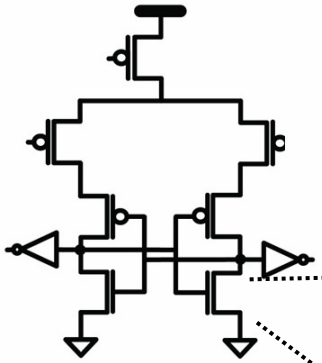

SONY
Cyber-shot
8.1

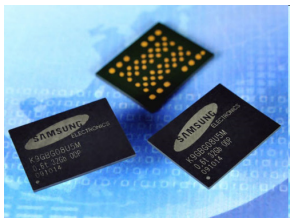


1-bit memory element (bitcell)
x 32 billion !! (for 32GB)

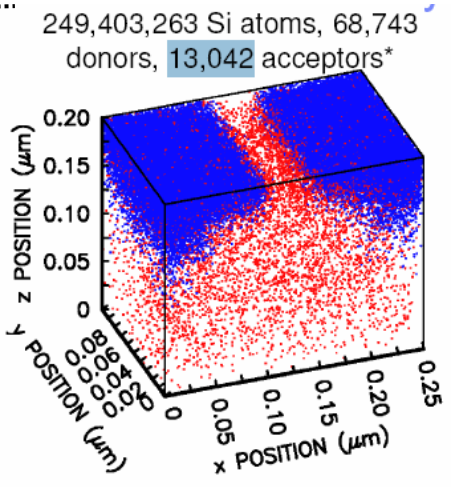
fab





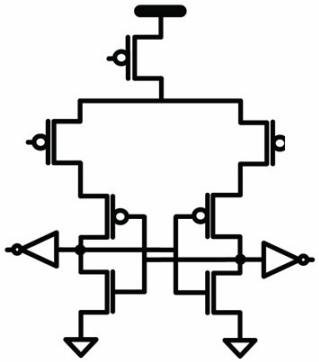


One transistor, in silicon



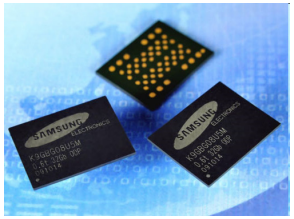
Random dopants cause performance variability, which affects yield

How to get fired!

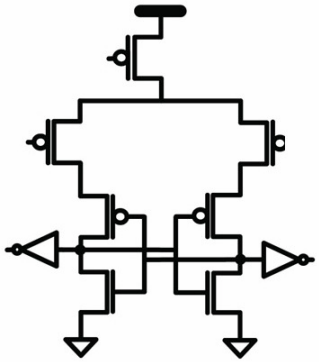


10% yield

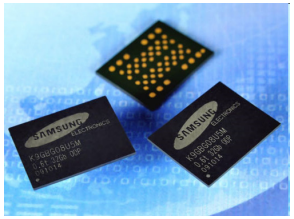
(10% of chips meet power and timing specifications)



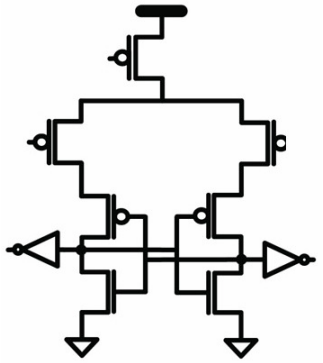
How to keep your job!

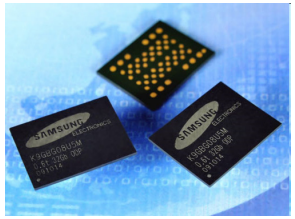


95% yield
(95% of chips meet specs)



How to keep your job!





95% yield
(95% of chips meet specs)

Therefore each bitcell must have yield of 99.999999% or higher (>6 sigma)

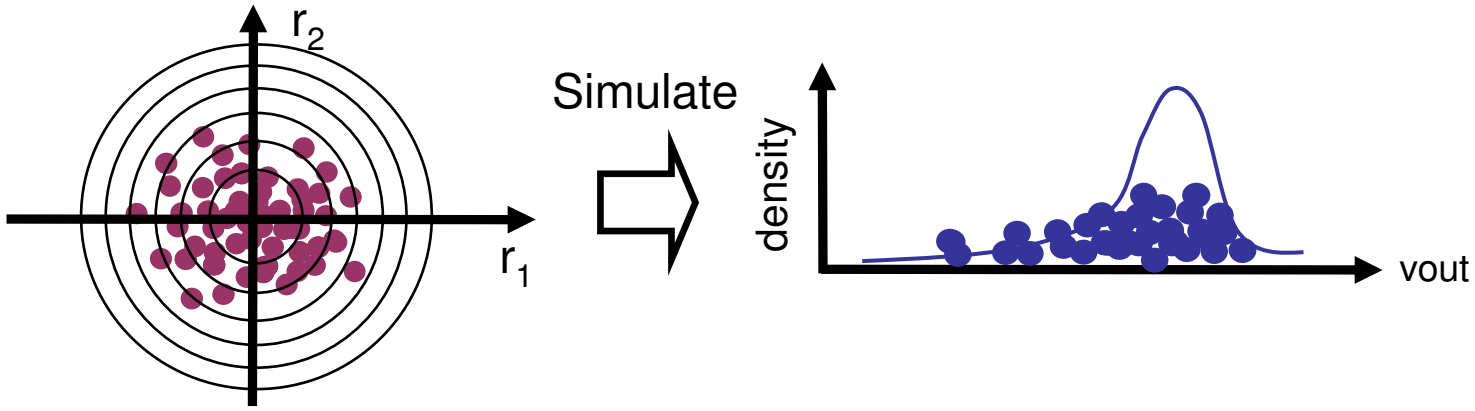
…How??

Ideal: not only measure yield, but
analyze tradeoff between yield & spec
i.e. symbolic density models

Samples of Tails

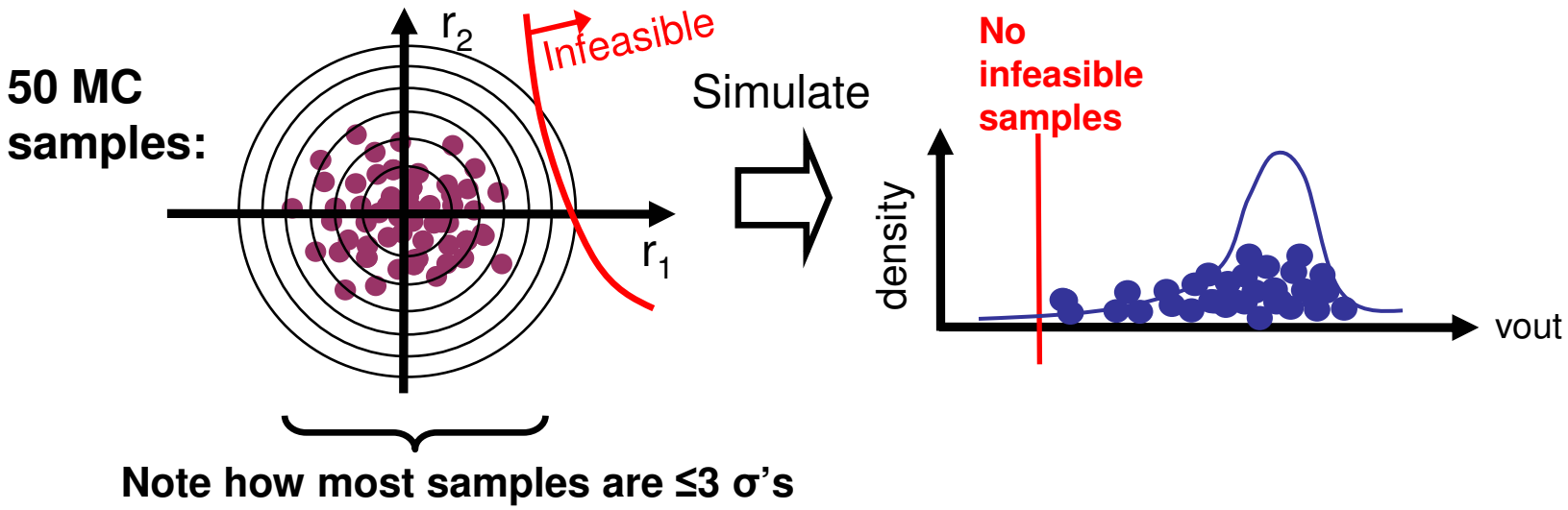
MC Sampling

- We have statistical models for the transistors
- We can simulate a bitcell circuit (using SPICE)
- Can combine these for “Monte Carlo” (MC) sampling



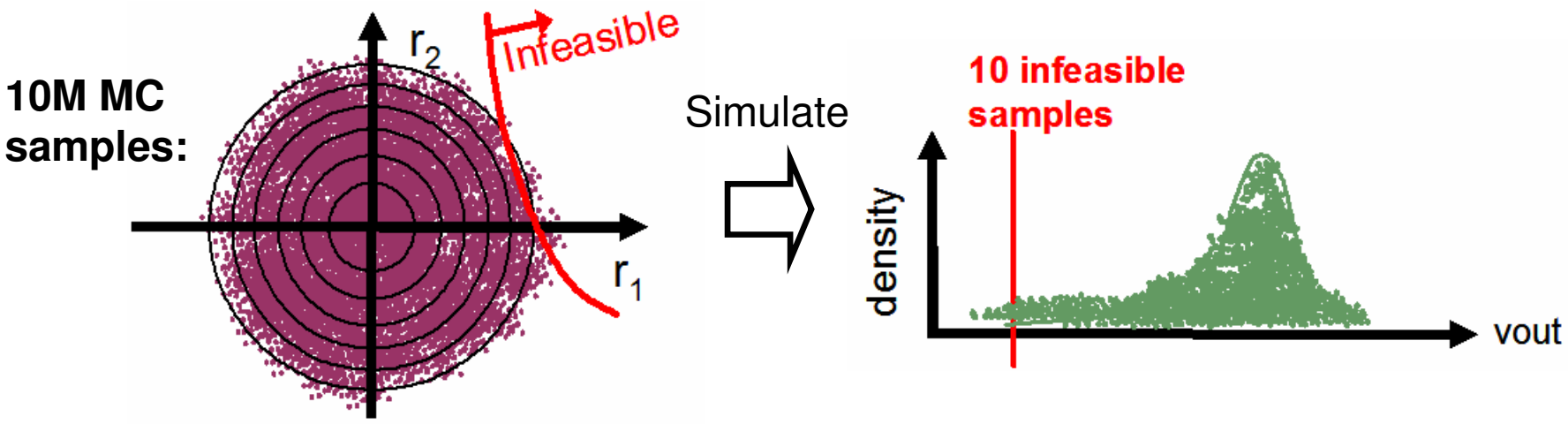
MC Sampling

- If 1/1M samples fails, then with a moderate number of samples we typically will see 0 failures
- E.g. 50/50 samples means 100% of simulations passed
- But we know yield is not 100%! (Argh!)



MC Sampling

- MC sampling would need on average 1M samples just to get 1 failure
- And for a decent yield estimate, want ≥10 failures
- Therefore would need ≥10M samples
  - Argh!!

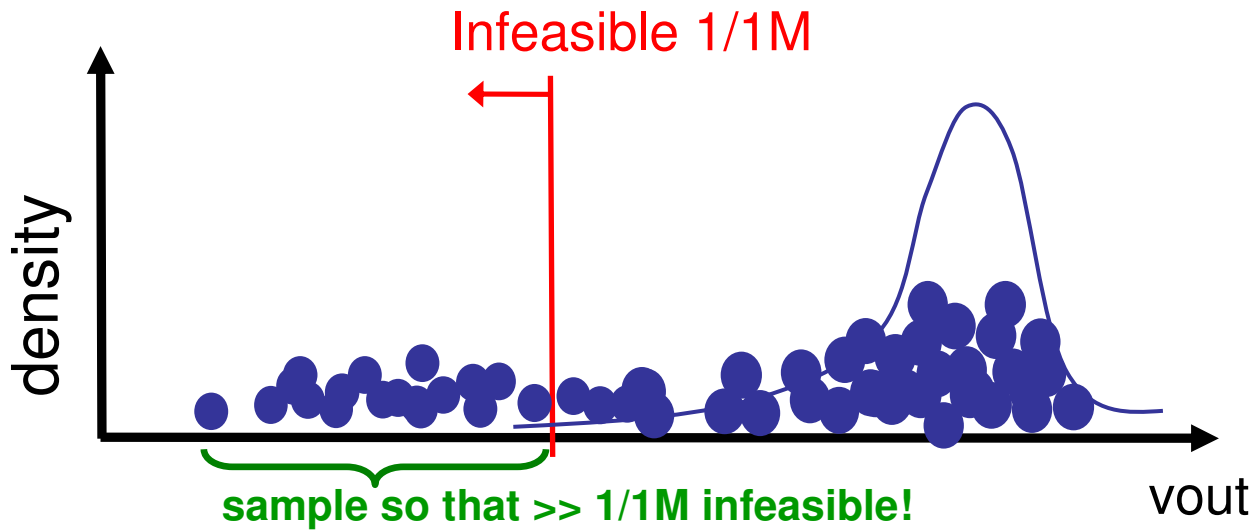


- ***Is there a practical alternative?***

Importance Sampling

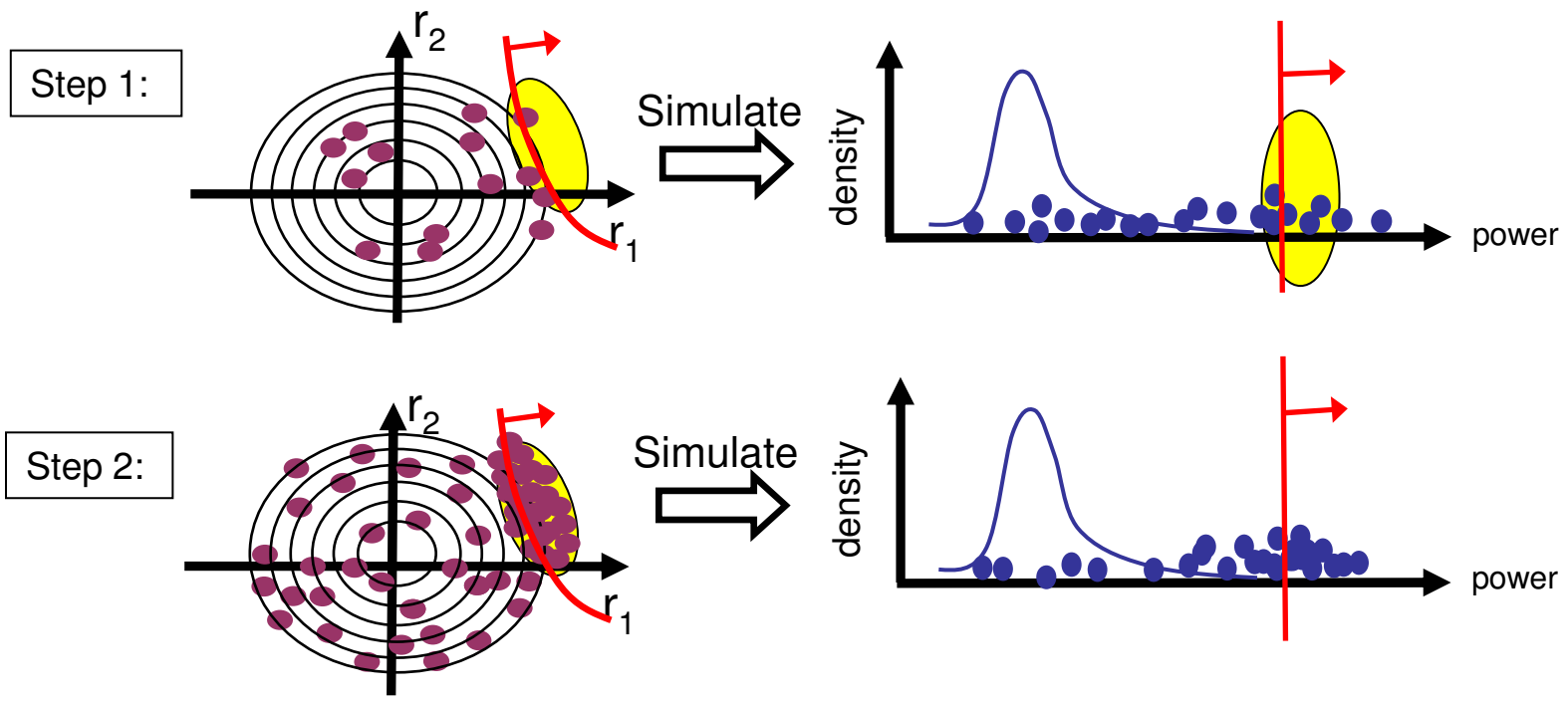
- We don't need to draw samples directly from the (true) distribution of process variations
- We just need to be *aware* of the distribution

- Instead, we can sample "wherever we want"
  - E.g. sample points with extremely low probability

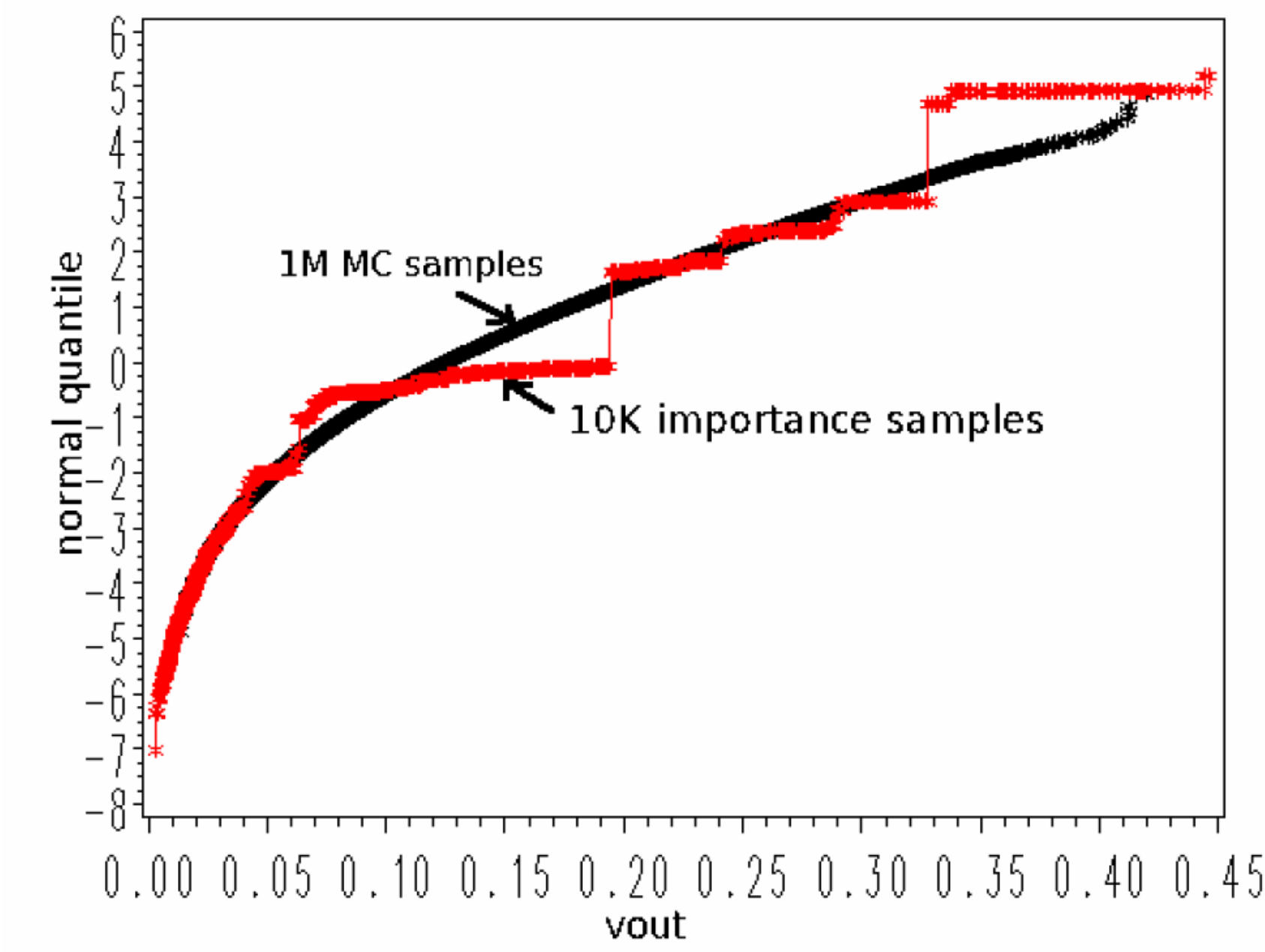


Importance Sampling

1. Find highest-probability regions of process space that cause infeas.
   - E.g. with evolutionary programming and SPICE-in-loop
2. Do a statistical sampling, but with sample pdf *biased to those regions*
   - Remember each sample's weight (= truepdf(r) / samplepdf(r))
3. Compute statistical estimates (e.g. extract pdf *somehow…*)



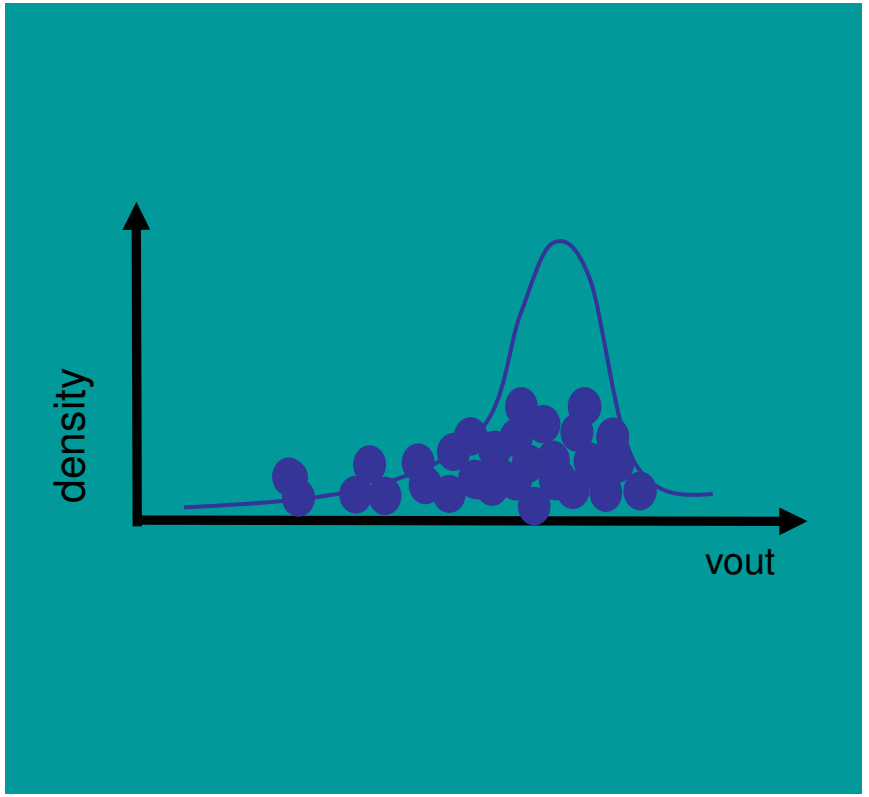
10K Importance samples vs 1M MC samples (bitcell)



Density Models

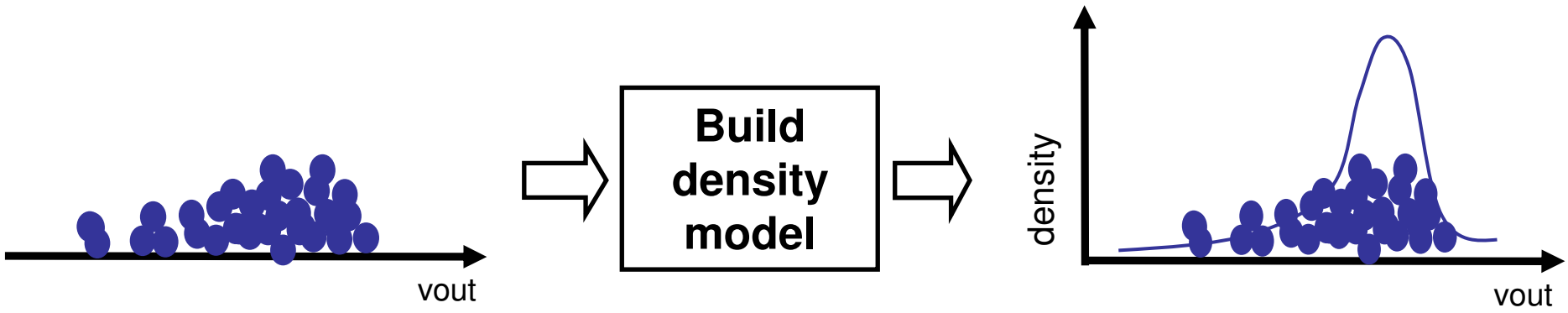
Disambiguation: Dense Models vs. Density Models

Dense model



Density model

Density Estimation

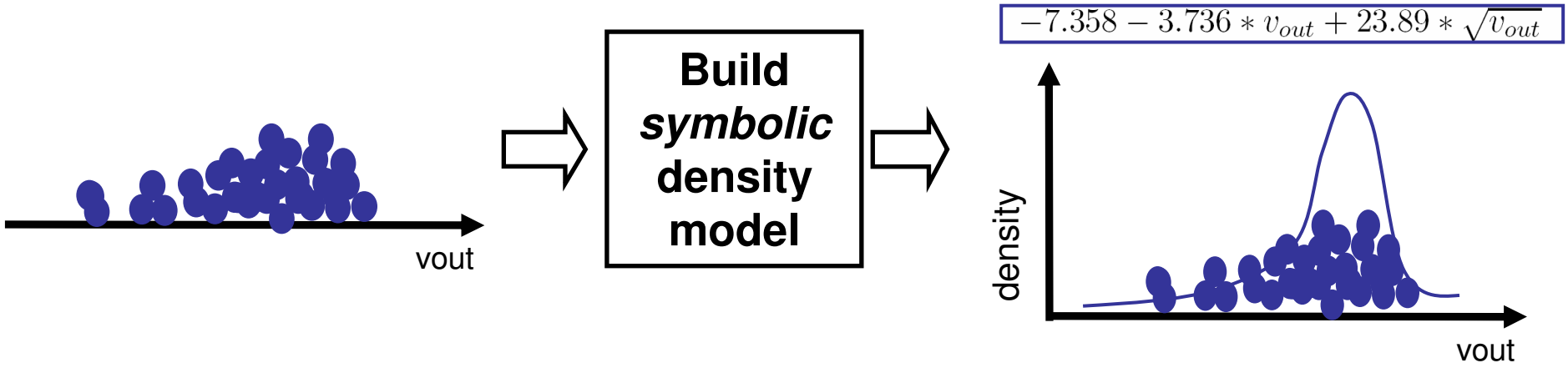


- Aims to maximize the likelihood of the input sample points
- Each point is treated equally (i.e. weights are the same, implicitly)

But…

- Not typically easy-to-analyze closed-form expressions
- How to handle importance-sampled data?

Symbolic Density Estimation



Aims:

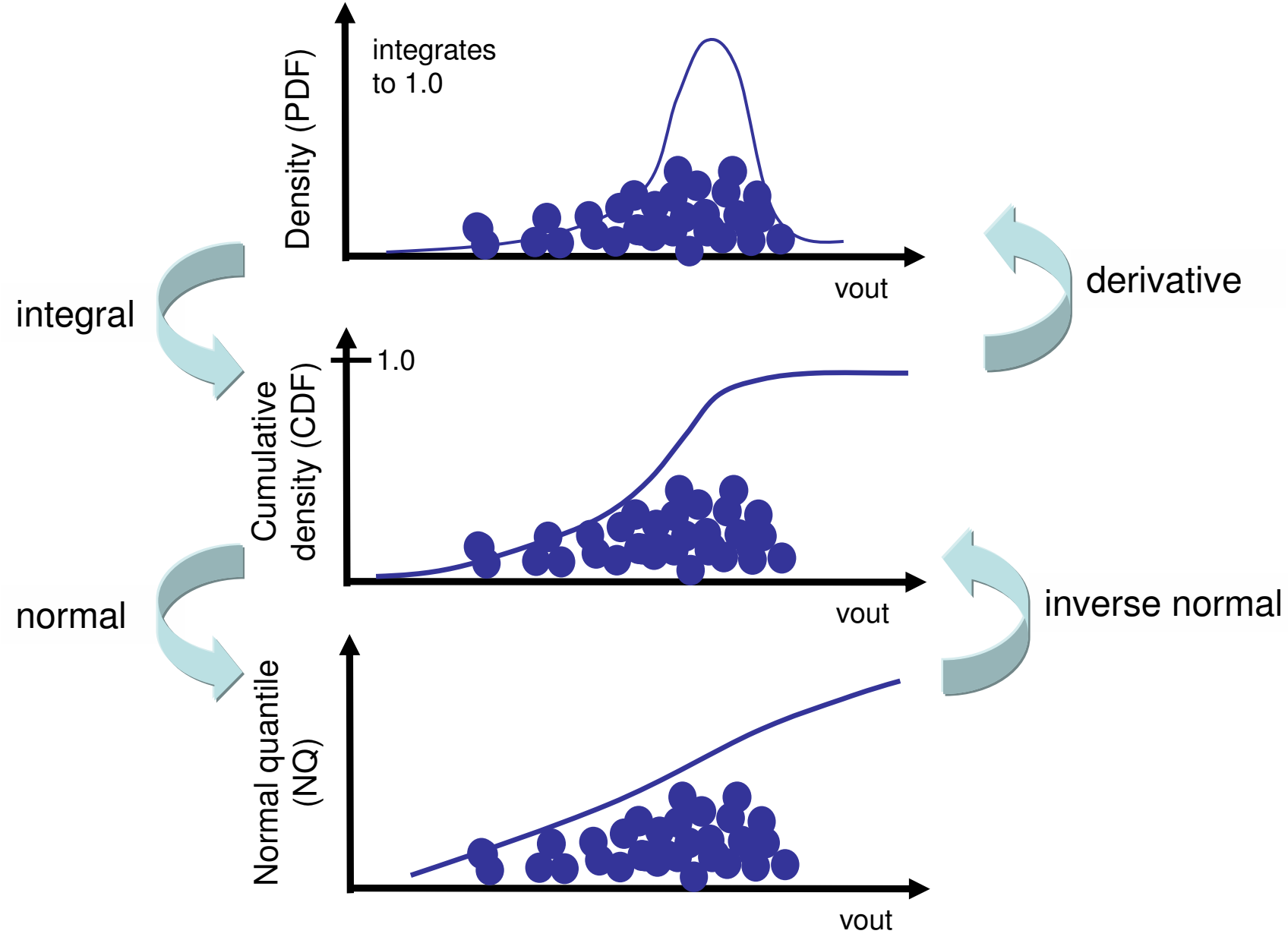
- Explicitly outputs easy-to-analyze closed-form expressions
- Handles importance-sampled data (i.e. weighted samples)

Key approach:

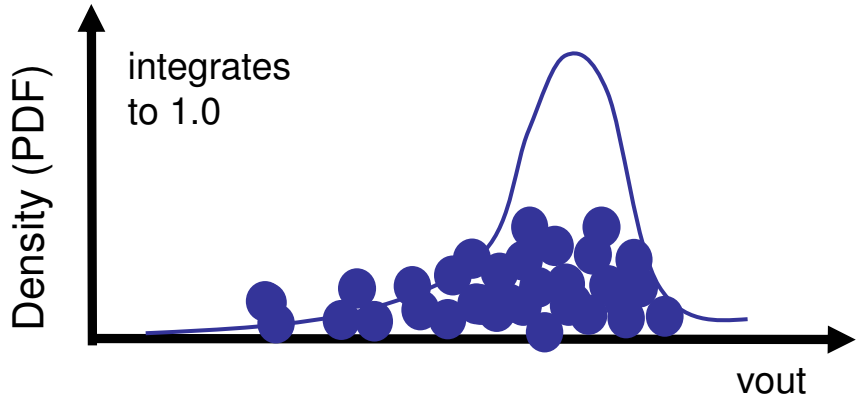
- Cast into a symbolic regression problem
- Apply genetic programming

Refresher: PDF, CDF, NQ

- Three different views of the same distribution



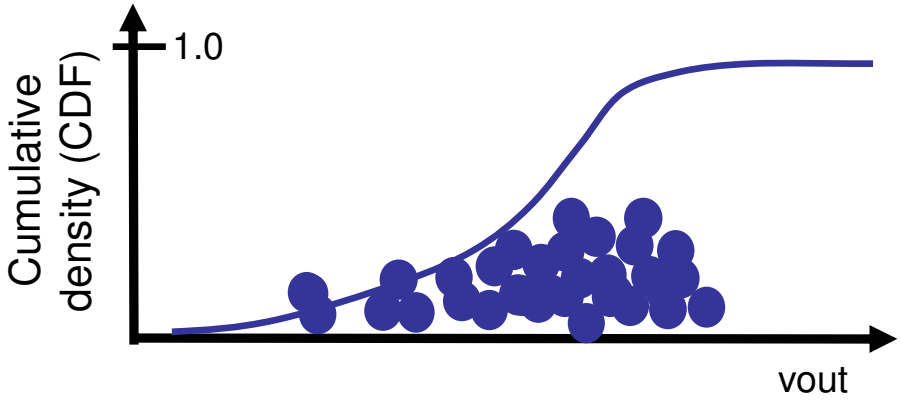
Model in PDF vs CDF vs NQ?



Constraint: integrate to exactly 1.0.

Strongly non-linear model

Constraint: model always ≥ 0.0

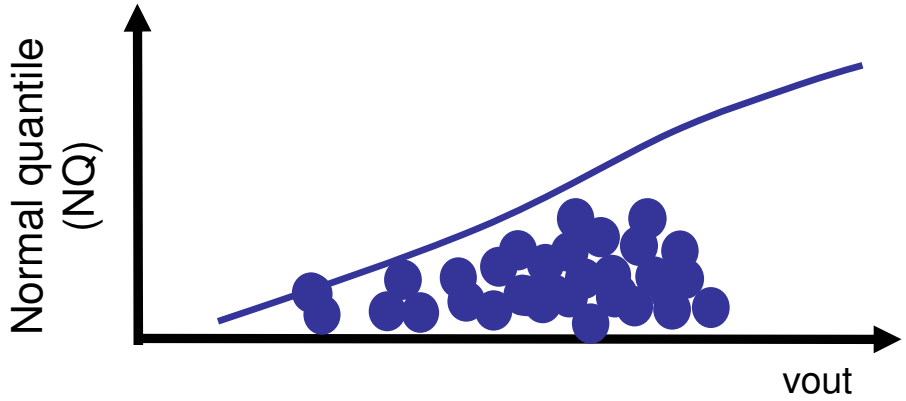


Constraint: model starts at 0.0, ends at 1.0. How to taper tails?

Weakly-nonlinear model

Constraint: model always ≥ 0.0

Constraint: monotonically increasing



Constraint: monotonically increasing

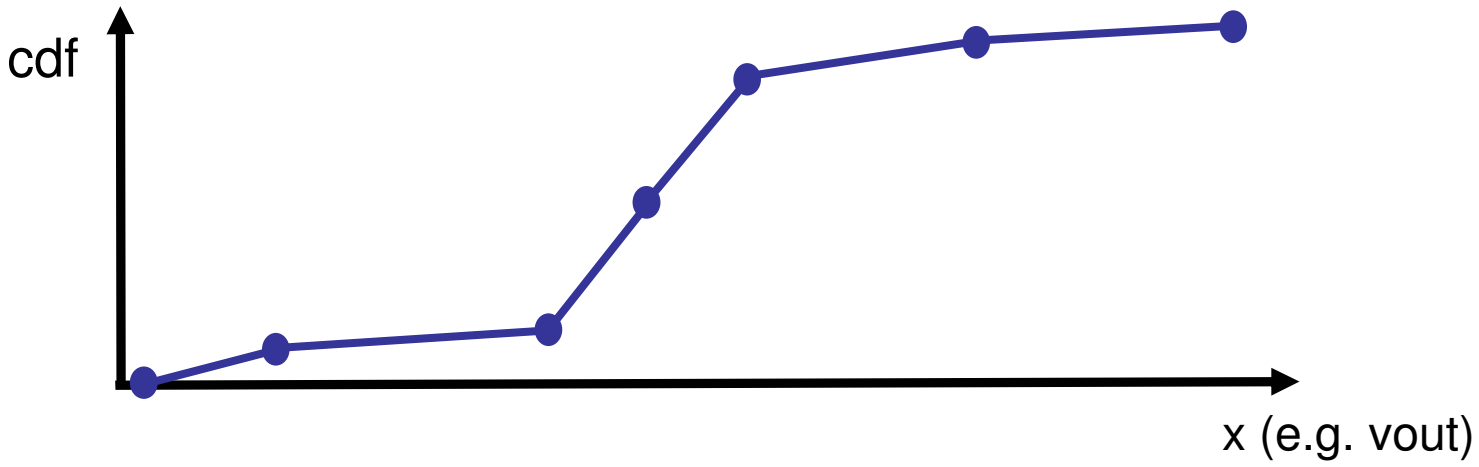
Nearly-linear model

Extrapolates nicely at tails

Symbolic Density Estimation

- Given importance sampled data: $\{x_i, w_i\}$
  - or plain MC data, just let w_i=1
- Assume w normalized

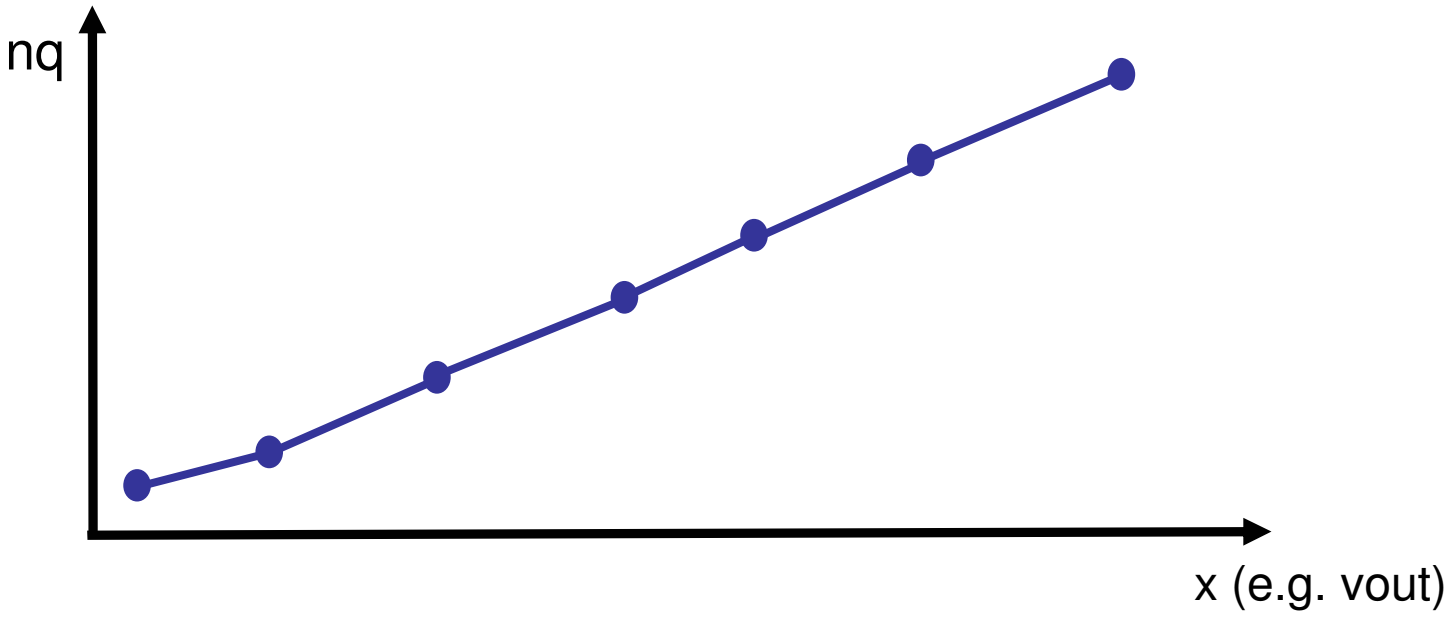
1. Sort x and w in order of ascending x
2. Compute numerical cdf: x→cdf
   - $cdf_i = \sum^{i}_{k=1} w_k$



Symbolic Density Estimation

Given IS or MC data (x_i, w_i)

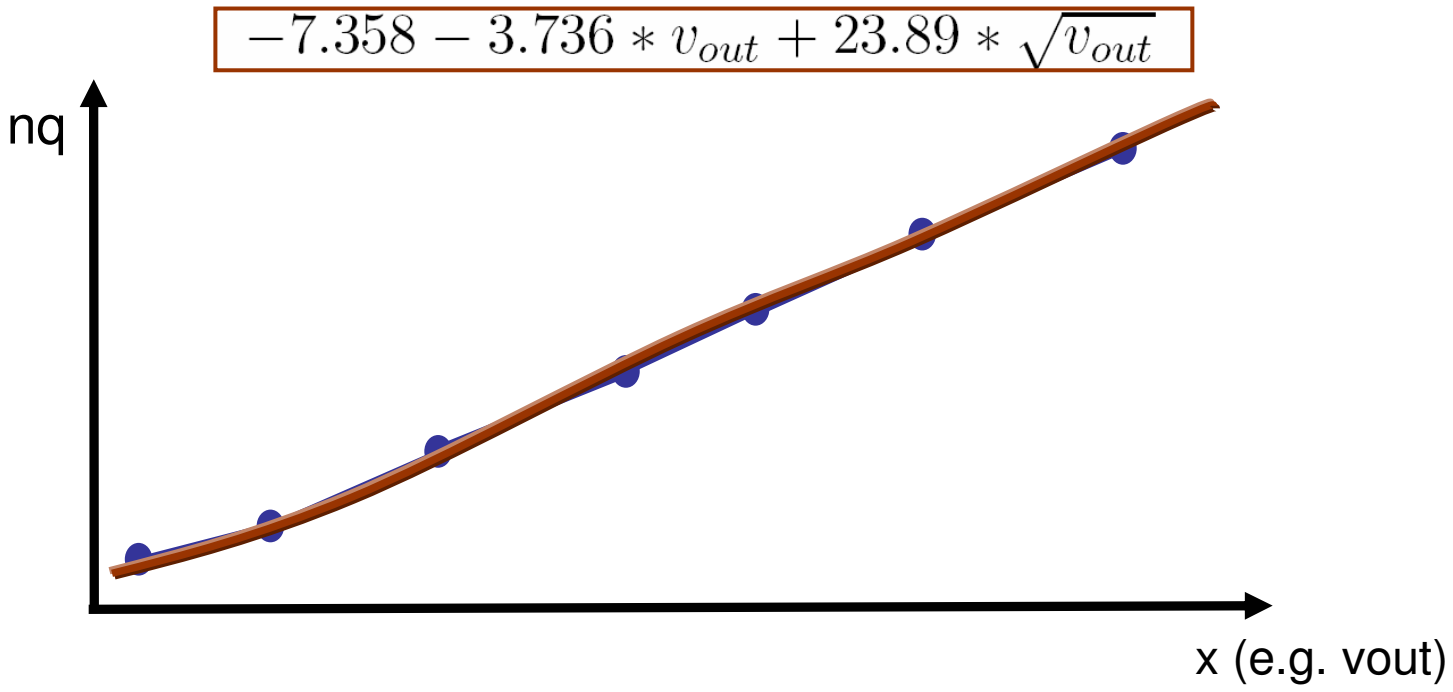
1. Sort in ascending x
2. Compute numerical cdf: x→cdf
3. Compute numerical normal-quantile: x→nq



Symbolic Density Estimation

- Given IS or MC data

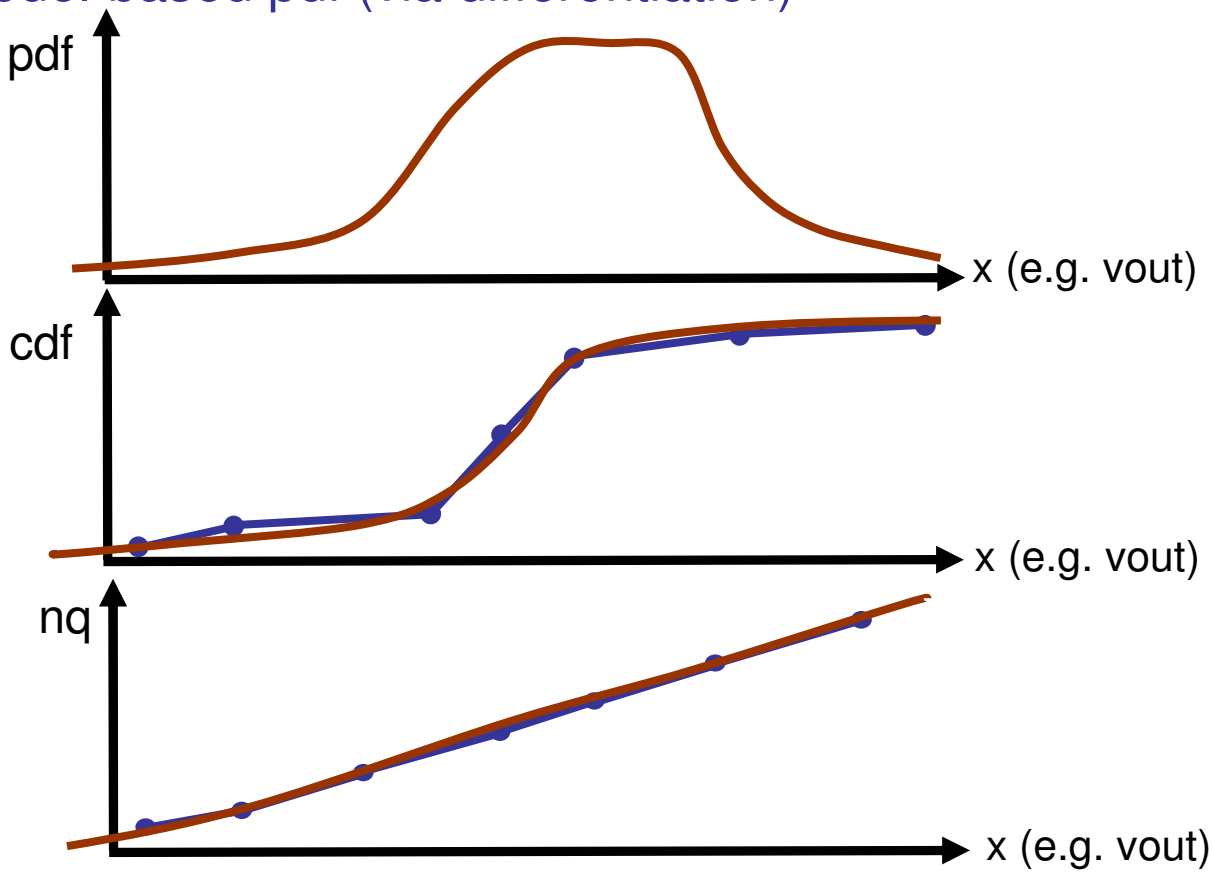
1. Sort in ascending x
2. Compute numerical cdf: x→cdf
3. Compute numerical normal-quantile: x→nq
4. With GP, find symbolic density model of x→nq



Symbolic Density Estimation

- Given IS or MC data

1. Sort in ascending x
2. Compute numerical cdf: x→cdf
3. Compute numerical normal-quantile: x→nq
4. With GP, find symbolic density model of x→nq
5. Compute model-based cdf (via inverse normal)
6. Compute model-based pdf (via differentiation)



Some GP Implementation Details

- Overfitting not a big issue because have 5000-50,000 samples and just 1d input. ("Trivial")
- Can use any GP symbolic regression system
- Can make it multi-objective, for a tradeoff between model complexity and model error

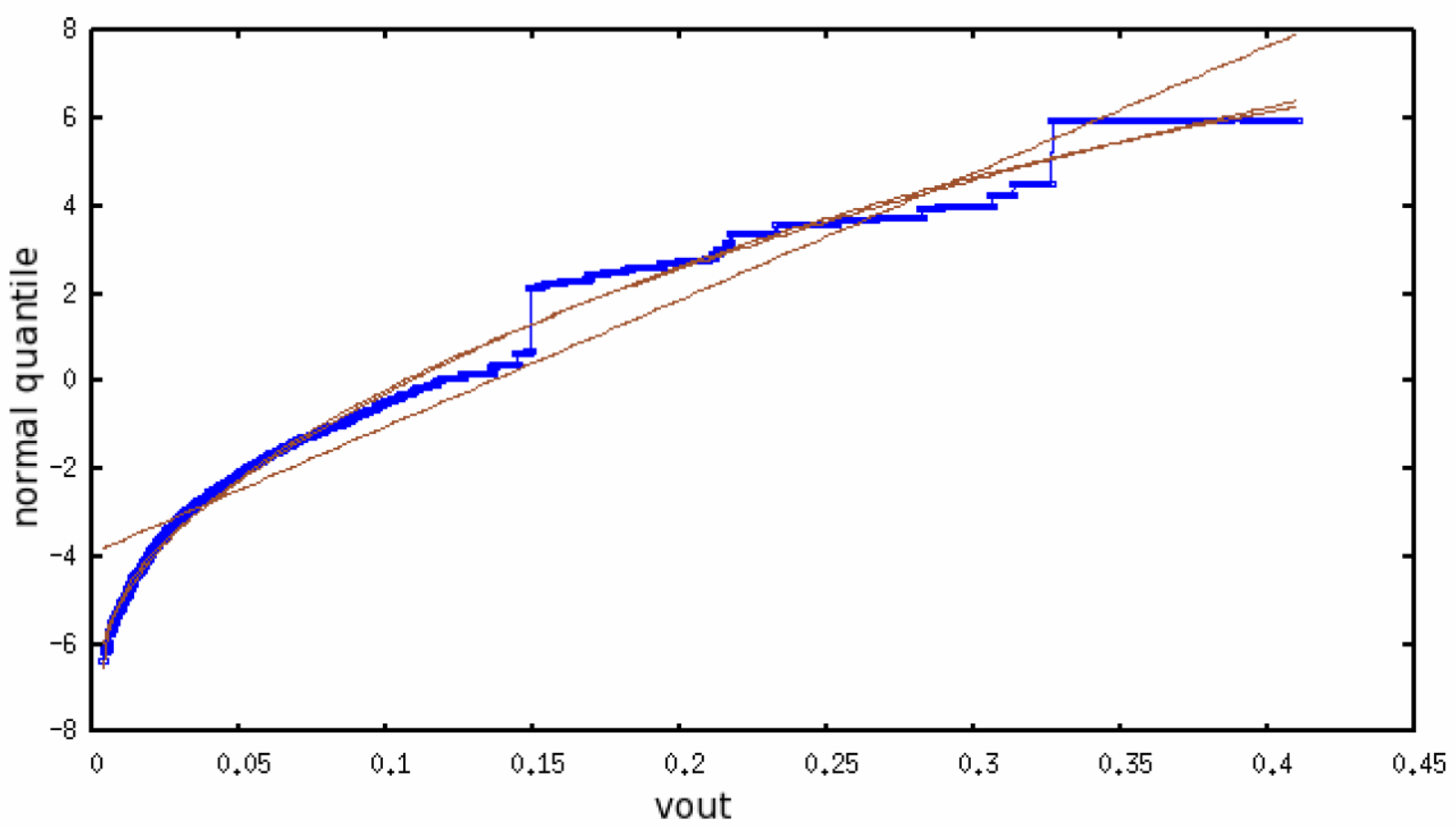
- I used CAFFEINE because:
  - It returns easy-to-interpret equations because search is constrained to canonical-form functions. Bloat not an issue.
  - It has built-in bias to working off linear models (optional)
  - It's multi-objective
  - It was convenient!

- For extra speed, I pre-pruned the data to 50 points:
  1. Took every n^{th} sample to get to 250 points
  2. Then applied SMITS balancing procedure to get to 50 points.
     - At each iteration, remove the sample that has lowest "deviation from linearity". Calculate deviation via local linear fits.

- I actually post-pruned the Pareto Front using SMITS too

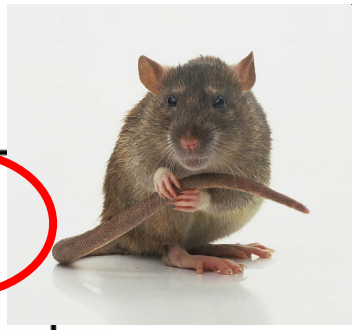
Results

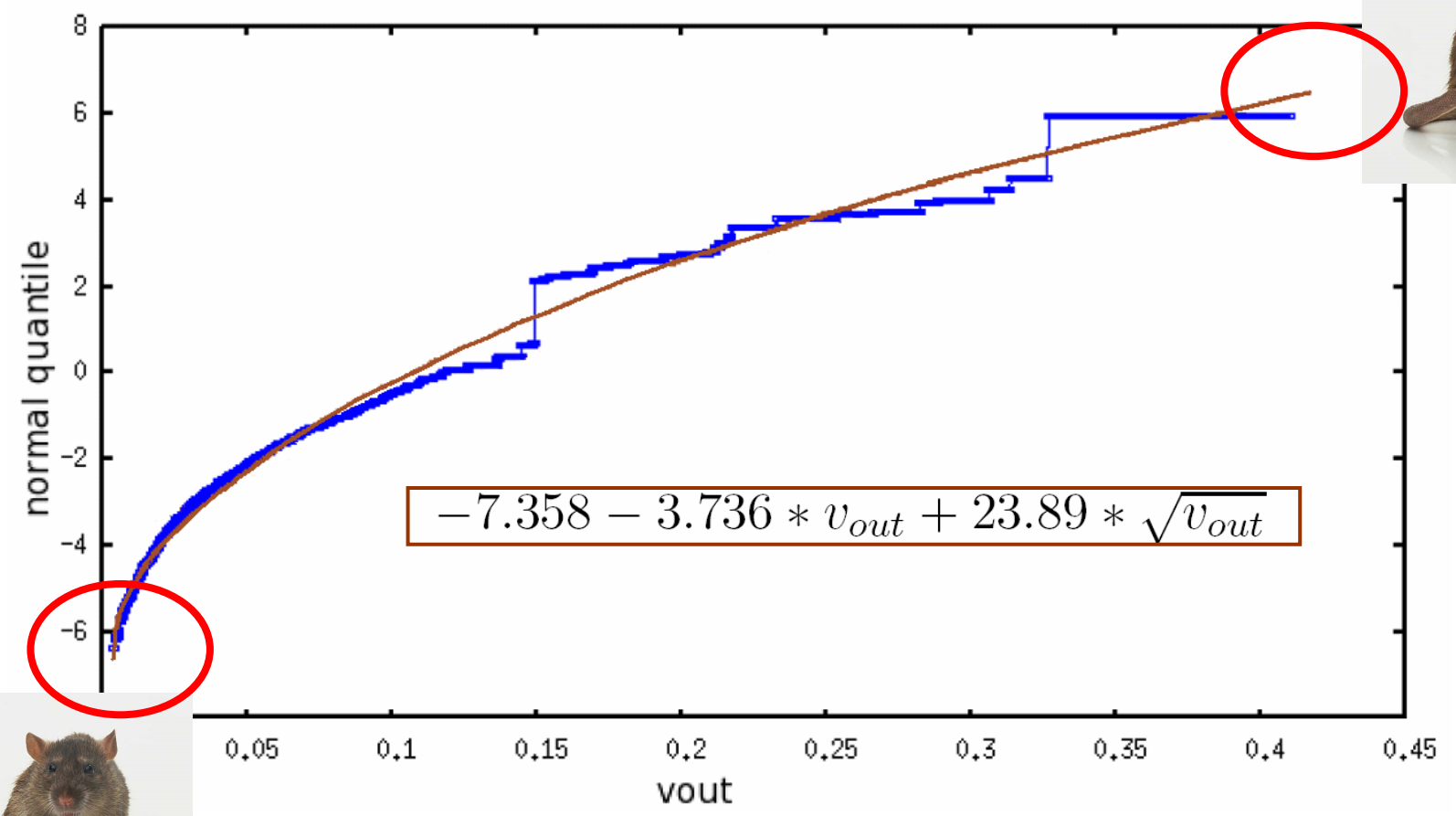
Results on bitcell: nq



Train error (%)	***nq* Expression**
26.3	$-3.903 + 28.90 * v_{out}$
11.42	$-7.358 - 3.736 * v_{out} + 23.89 * \sqrt{v_{out}}$
11.01	$368.1 - 46.482 * v_{out} + 170.959 * \sqrt{v_{out}}$ $+41.0435 * log10(1.07285e - 7 * \sqrt{v_{out}})$ $-0.002472/\sqrt{max(0, 1.562e - 10/\sqrt{v_{out}})}$

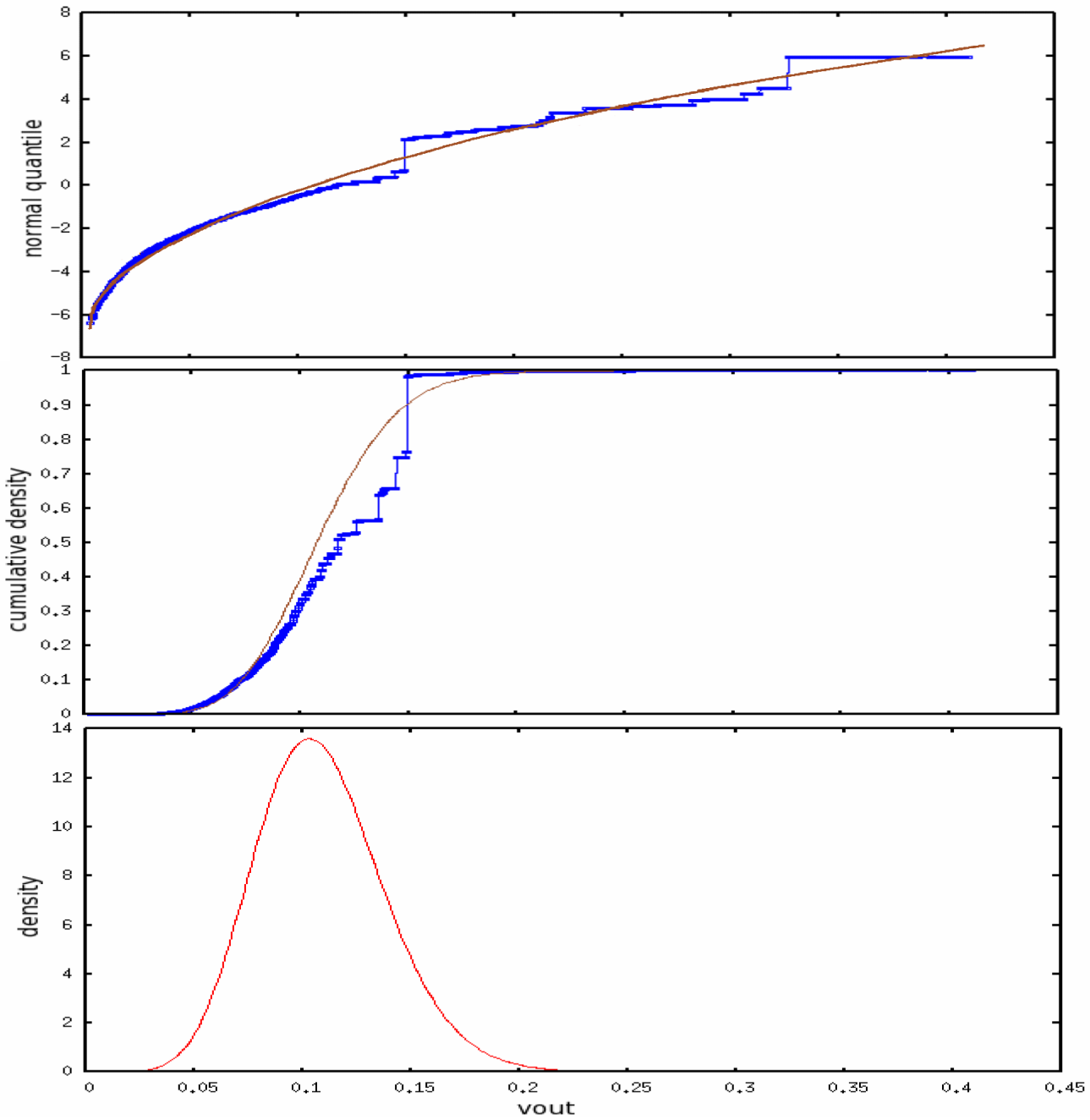
Note the tails – extrapolation looks good





This is a valuable result for memory circuit designers.

Results: nq, cdf, pdf



Conclusion

Take-Away Lessons for GPers

1. Symbolic Density Modeling

- We can apply SR to density modeling: symbolic density modeling
  - Modeling in nq space greatly simplifies the problem
  - Importance sampling enables models of one-in-a-billion tails
  - Demonstrated this on a real-world problem (memory design)
  - Many other real-world problems in density modeling…

2. On GP Real-World Applications

- One recipe for successful industrial application is: give GP problems that are trivial for GP!
  - 1-D SR and 50 samples is trivial. This is a good thing because…
  - …GP can then quickly & reliably get high-quality results!!
  - Just because problems are trivial for GP doesn't mean the overall value is trivial – my memory design example means $$.

3. Tails

- Are funny☺

That's All Folks!