# Bayesian Global Optimization and Knowledge Extraction for Active Predictive Analytics

# Trent McConaghy Co-Founder & CTO Sociedo

# Outline

- Bayesian Global Optimization (BGO) "Push designs"
  - Motivating application / what computer chips
  - Challenges confidence intervals, scalability
  - Applications Deep Belief Nets, Ads, Health, more

- Active Predictive Analytics "Pull designs"
  - Fundamental optimization issue loss of control & insight
  - Using Active Learning for Fast Combination Sweep
  - Using Predictive Analytics / Knowledge Extraction
  - Applications like opt, plus more

Global Optimization Motivating Example: Circuit Verification

# **Motivating Example: Circuit Verification**



# **Circuit Verification: General Idea**

**Cast verification as a global optimization problem:** 

- Search through space of "corners" x
- Minimize / maximize simulated output value f(x)

Then, solve the optimization problem reliably.



# **Circuit Verification: Global Optimization**

Cast verification as a global optimization problem:

- Search through space of "corners" x
- Minimize / maximize simulated output value f(x)

Then, solve the optimization problem reliably.

#### Attributes of a good global optimizer:

- Min # evaluations
- Reliably finds global optimum

**Options:** 

- Deterministic: branch and bound, interval methods, ..
- Stochastic: SA, EP/ES, CMA-ES, PSO, ACO, ...
- Local optimization with restarts
- Recast to convex, then Geometric Programming

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### **Circuit Verification: Bayesian Global Optimization**

Cast verification as a global optimization problem:

- Search through space of "corners" x
- Minimize / maximize simulated output value f(x)

Then, solve the optimization problem reliably.

#### **Option: Bayesian [Global] Optimization**

Original ref: Jonas Mockus, "On Bayesian Methods for Seeking the Extremum," Optimization Techniques, 1974, pp. 400-404

My fave ref..



Journal of Global Optimization 13: 455–492, 1998. © 1998 Kluwer Academic Publishers. Printed in the Netherlands.

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#### Efficient Global Optimization of Expensive Black-Box Functions

DONALD R. JONES<sup>1</sup>, MATTHIAS SCHONLAU<sup>2,\*</sup> and WILLIAM J. WELCH<sup>3,\*\*</sup> <sup>1</sup>Operations Research Department, General Motors R&D Operations, Warren, MI, USA; <sup>2</sup>National

#### BGO Walk-Through - Step 1 Initial sampling, simulate, build model, choose x<sub>new</sub>



#### BGO Walk-Through Step 2 Simulate new point, update model, choose x<sub>new</sub>



## **BGO Underlying Model**



- Typically a Gaussian Process Model (GPM)
  - Natural interpolator
  - Convenient confidence intervals
  - Well-behaved, no crazy extrapolation (usually)

### **BGO for Circuit Verification – Example Convergence**



## **BGO: When to stop?**

Want min # evals, and to "guarantee" global optimum. Extreme cases:

- If run all points: guarantees find best, but no speedup
- If run 0 points, it guarantees speed, but never finds best Baseline approach:



### **BGO: When to stop?**

#### **Revised stopping criterion: "let confidence intervals stabilize"**

- Stop if (max evaluated value) is constant for ≥ N1 evaluations;
- and (predicted upper bound) < (max evaluated) for  $\geq N2$  evals
  - Where N1, N2 are a function of "prediction difficulty" = (1.0 rank r)
  - Where rank r is Spearman Rank Correlation between predicted & actual



#### BGO: Representative Convergence Curve -most look like this

buffer\_chain\_mm\_avgdly4\_5\_CRTSN: Max y Convergence. r=0.901. Simulated 79/1080 = 7.31% of full-fact. Speedup=13.67



### BGO: Representative Convergence Curves -some look like this

noise\_margin\_2\_NMGoalValue\_CRTSN: Max y Convergence. r=0.949. Simulated 179/3240 = 5.52% of full-fact. Speedup=18.10



#### **BGO: Convergence Curves** -even "difficult" curves converge



# Samples

# **BGO: "Difficult" Convergence Curve**



# **BGO: "Difficult" Convergence Curve**

![](_page_17_Figure_1.jpeg)

# **BGO for Circuit Verification – VCO of PLL**

![](_page_18_Figure_1.jpeg)

#### **BGO Benchmarks on 226 Circuit Verification Problems**

- 226 test cases in benchmark suite:
  - From Solido customers, in-house realistic cases, and in-house corner cases targeting challenging problems
  - Many contain complex interactions, non-linearities, discontinuities, etc.
- 226/226 (100%) of cases find true optimum
- Speedup 2.34X to 226X
- Median speedup is 22X

![](_page_19_Figure_7.jpeg)

# **BGO Scalability Challenge: # Samples**

![](_page_20_Figure_1.jpeg)

- Problem: GPM training is O(N<sup>3</sup>) on # Training Samples
- Becomes very unhappy when >1000 samples
- This happens for circuit verification problems with larger # dimensions and highly nonlinear circuit
- First solution: just cut loose and sim all
- Is there a better way?

## **Solution: Divide-and-Conquer on Training Samples**

![](_page_21_Figure_1.jpeg)

- New model is a *set* of Gaussian Process Models (GPM)
- One GPM for each region of input x space
- Regions are automatically determined at build time
  - Via classic CART learning
  - Stop at a leaf when <700 samples</p>
- Build a GPM on each leaf's samples (and k neighbors)
  - Each GPM is O(1) on # training samples because N=const
  - CART learning is O(N log N) on # samp with tiny constant

### **Benchmarking: GPM vs Divide-and-Conquer GPM**

					GPM		Divide	-and-cor GPM	nquer
	#	# train	# test	Build	Test		Build	Test	
Problem	vars	pts	pts	Time (s)	Time (s)	Error	Time	Time	Error
Low-dimensional									
opamp-pvt-bandwidth	10	4425	1475	667.4	91.1	0.044	55.6	7.4	0.006
opamp-pvt-dc_gain	10	4425	1475	741.9	91.5	0.001	57.9	8.5	0.003
opamp-pvt-gain_margin	10	4425	1475	319.9	92.2	0.313	59.6	8.2	0.168
opamp-pvt-gbw	10	4425	1475	845.7	92.8	0.010	62.4	8.8	0.008
opamp-pvt-idc	10	4425	1475	775.2	91.7	0.000	41.2	8.2	0.000
opamp-pvt-phase_margin	10	4425	1475	268.2	90.9	0.149	49.8	6.6	0.155
High-dimensional									
senseamp_pwr	125	3750	1250	failed	failed	failed	165.8	37.9	4.139
opamp_AV	215	600	200	38.3	18.2	2.933	23.3	9.8	3.628
opamp_SR	215	600	200	34.8	18.2	2.604	37.3	9.5	2.515
compar_bw	639	1502	500	246.2	56.9	16.010	73.7	23.0	16.458
opamp_PM	215	600	200	63.9	18.3	3.678	26.7	9.4	2.441
opamp_BW	215	600	200	34.9	18.3	1.800	31.6	9.6	2.084
mem	385	7500	2500	failed	failed	failed	422.4	78.3	0.480
senseamp_delay	125	3750	1250	failed	failed	failed	286.0	38.4	5.135

# Bayesian Global Optimization For ML Modeling

### **BGO for Deep Belief Net (DBN) Optimization**

![](_page_24_Figure_1.jpeg)

Whole model		Per-layer		
Parameter	Prior	Parameter	Prior	
pre-processing	raw or ZCA	n. hidden units	$\log U(128, 4096)$	
ZCA energy	U(.5, 1)	W init	$U(-a,a)$ or $\mathcal{N}(0,a^2)$	
random seed	5 choices	a	algo A or B (see text)	
classifier learn rate	$\log U(0.001, 10)$	algo A coef	U(.2,2)	
classifier anneal start	$\log U(100, 10^4)$	CD epochs	$\log U(1, 10^4)$	
classifier $\ell_2$ -penalty	$0 \text{ or } \log U(10^{-7}, 10^{-4})$	CD learn rate	$\log U(10^{-4}, 1)$	
n. layers	1 to 3	CD anneal start	$\log U(10, 10^4)$	
batch size	20 or 100	CD sample data	yes or no	

### **BGO DBN Optimization, on Convex Dataset**

![](_page_25_Figure_1.jpeg)

[Bergstra et al NIPS 2011]

#### **BGO DBN Optimization, on MNIST Dataset**

![](_page_26_Figure_1.jpeg)

### **BGO ConvNN Optimization on CIFAR-10 data**

![](_page_27_Figure_1.jpeg)

[Snoek et al NIPS 2012]

### Some Applications of Global Optimization

- Semiconductors: Verify a circuit across PVT corners
- ML modeling: Find optimal model meta-parameters (DeepNN, RF, SVM, ..), for application to computer vision etc.
- Health: protein shape prediction (minimal energy configuration)
- Business Intelligence: optimize churn & other key performance indicators (KPIs)
- Big data infrastructure: optimize reliability / uptime, minimize power consumption, ..
- Internet / mobile: auto SEO, optimize for app store placement (rank, profitability)
- Oil & gas: capital & resource allocation
- Space: minimize interplanetary trajectory flight time

# Active Predictive Analytics / Supercharged Manual Design

# Automated vs. Manual Design

#### Benefits of Automated (Optimization):

- Efficient; high throughput
- Scalable no human bottleneck
- Optimal design (assuming you measure everything, and have appropriate objectives & constraints)

### Benefits of Manual:

- Retain insight
- Retain control
- Insight & control lead to creative structural improvements
- Don't need perfect measures, objectives, constraints
- Familiar

## Automated vs. Manual Design

#### **Benefits of Automated:**

• Efficient, optimal, ..

#### Benefits of Manual:

• Retain insight, control, ..

#### Q: Can we get the best of both worlds?

- Can we "supercharge" manual design?
- What might that mean?

### **Background: "Sweep" in Manual Design**

![](_page_32_Figure_1.jpeg)

### **Background: "Sweep" in Manual Design**

	1d Perturb	1d Sweep	Combination Sweep
Fast	Yes	Yes	No (takes nval <sup>nvar</sup> sims)
Accurate	NO (too local, misses interactions)	No (misses interactions)	Yes
Scalable	Yes	Yes	No (≤5 vars)
	x2 x1 f(x) x1 x2 x2	x2 x1 f(x) x1 x2 x2	f(x)

### **Consider: Combination Sweep in >>1 Dimension**

![](_page_34_Figure_1.jpeg)

Insight & control would be perfect! But two big problems:

- **1. Computational expense is insane** (e.g. 10G sims for 10 variables and 10 values per variable)
- 2. How to visualize beyond 3d? (and 3d is weak too)

#### Problem #1: Insane Computational Expense Solution: Active Learning (via BGO)

	1d Perturb	1d Sweep	Combination Sweep	Active Learning (Fast Comb. Sweep)
Fast	Yes	Yes	No (takes nval <sup>nvar</sup> sims)	Yes
Accurate	NO (too local, misses interactions)	No (misses interactions)	Yes	Yes
Scalable	Yes	Yes	No (≤5 vars)	Yes
	x2 x1 f(x) x1	x2 x1 f(x) x1 x1 x2 x2	f(x)	Adaptively choose evaluations, predict the rest

#### Problem #2: How to Visualize Sweeps in >>1 Dimension? Solution: Predictive Analytics / Knowledge Extraction

![](_page_36_Figure_1.jpeg)

### The key: focus on *important* variables and interactions

- Calculate impacts of x<sub>i</sub>, x<sub>i</sub>&x<sub>i</sub> by mining the GPM
- Show impacts
- Let user select an x<sub>i</sub> or x<sub>i</sub>&x<sub>i</sub>
- Show mapping from  $x_i$ ,  $x_i \& x_j$  to f(x)

#### Problem #2: How to Visualize Sweeps in >>1 Dimension? Solution: Predictive Analytics / Knowledge Extraction

![](_page_37_Figure_1.jpeg)

#### Problem #2: How to Visualize Sweeps in >>1 Dimension? Solution: Predictive Analytics / Knowledge Extraction

(ID	)=0)	True	3.928	53.50p		137.би			- 3.586u -	
est (1) (ID	)=-1)	False	2.469	6.482p		64.28u		M	- 3.191u -	
est (2) (ID	)=-б)	False	2.379	6.722p		65.89u		E	- 2.796u -	
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28.79%	opar	mp.xcm1_m1.w	2.500u						5.960u -	
) 12.74%	opar	mp.xcm1_m1.l,	300.0n, 2.500u					_	3.586u -	
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	Se Va	elect in ariable	itera s	cti	ng		AllVar	opamp.xcm1	2.796u - 2.500u 2.204u - 1.809u 1.414u - 1.020u - 625.0n - 4.375u - 3.980u - 3.586u -	x 258 2n Videlue: 4.411u
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	Se Va	elect in ariable	itera s	cti	ng		n AllVar	L_m1.w opamp.xcm1	2.796u - 2.500u 2.204u - 1.809u 1.414u - 1.020u 625.0n - 4.375u - 3.980u 3.586u - 3.586u - 3.191u - 2.796u -	x 258 2n y (dlue): 4.411u
	Se Va	elect in ariable	itera s	cti	ng		ranin AllVar	xcm1_m1.w	2.796u - 2.500u 2.204u - 1.809u 1.414u - 1.020u - 625.0n - 4.375u - 3.980u - 3.586u - 3.191u - 2.796u - 2.500u -	x 258 2n y Volue: 4.411u
	Se Va	elect in ariable	itera s	cti	ng		Branin AllVar	mp.xcml_ml.w	2.796u - 2.500u - 2.204u - 1.809u - 1.414u - 1.020u - 625.0n - 4.375u - 3.980u - 3.586u - 3.586u - 3.191u - 2.796u - 2.500u - 2.204u -	x 258 2n y (value): 4.411u
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В

### **Problem #2.1: How to Choose Slicing Plane?**

Example: Plotting f(x) vs  $x_1 \& x_2$ , so need to fix the value for  $x_3$ .

![](_page_39_Figure_2.jpeg)

### **Problem #2.1: How to Choose Slicing Plane?**

Generalization: Need to fix the value for *all vars. not being plotted*.

![](_page_40_Picture_2.jpeg)

#### Problem #2.1: How to Choose Slicing Plane? Solution: User Interactively Fixes the Value for "Current x"

![](_page_41_Figure_1.jpeg)

#### Problem #2.1: How to Choose Slicing Plane? Solution: User Interactively Fixes the Value for "Current x"

Sindlated	COSC	Area	AllVa
True	3.928	53.50p	137.6u
True	1.831	3.418p	34.52u
True	3.928	53.50p	137.6u
False	2.469	6.482p	64.28u
False	2.379	6.722p	65.89u
	True True True False False	True 3.928   True 1.831   True 3.928   False 2.469   False 2.379	True 3.928 53.50p   True 1.831 3.418p   True 3.928 53.50p   False 2.469 6.482p   False 2.379 6.722p

#### Save net Select interacting ... Simulate

#### Impacts variables

Impact	ariables	Value (Explore)
358.48%	opamp xcm1_m1.l	300.0n
28.79%	opamp cm1_m1.w	2.500u
12.74%	opamp.xcm1_m1.l, opamp.xcm1_m1.w	300.0n, 2.500u

![](_page_42_Figure_5.jpeg)

# Demo

# Active Learning + Predictive Analytics

= Active Predictive Analytics (aka Supercharged Manual Design)

#### Has Benefits of Automated (Optimization):

- Efficient; high throughput
- Scalable no human bottleneck
- Optimal design

#### And Has Benefits of Manual:

- Retain insight. Actually, even better insight than before!
- Retain control
- Insight & control lead to creative structural improvements
- Don't need perfect measures, objectives, constraints
- Familiar "just" sweeps

### **Applications of Active Predictive Analytics** *include* **Applications of Global Optimization**

- Semiconductors: Verify a circuit across PVT corners
- ML modeling: Find optimal model meta-parameters (DeepNN, RF, SVM, ..), for application to computer vision etc.
- Health: protein shape prediction (minimal energy configuration)
- Business Intelligence: optimize churn & other key performance indicators (KPIs)
- Big data infrastructure: optimize reliability / uptime, minimize power consumption, ..
- Internet / mobile: auto SEO, optimize for app store placement (rank, profitability)
- Oil & gas: capital & resource allocation
- Space: minimize interplanetary trajectory flight time

### Some Users of Bayesian Global Optimization or Active Predictive Analytics

![](_page_46_Picture_1.jpeg)

![](_page_46_Picture_2.jpeg)

![](_page_46_Picture_3.jpeg)

![](_page_46_Picture_4.jpeg)

![](_page_46_Picture_5.jpeg)

![](_page_46_Picture_6.jpeg)

![](_page_46_Picture_7.jpeg)

![](_page_46_Picture_8.jpeg)

![](_page_46_Picture_9.jpeg)

![](_page_46_Picture_10.jpeg)

![](_page_46_Picture_11.jpeg)

![](_page_46_Picture_12.jpeg)

# Conclusion

### "Push Designs": Bayesian Global Optimization

- Application to circuit verification, ML modeling
- How to handle CIs that lie, and scaling up

#### alue): 4.411u/

#### "Pull Designs": Active Predictive Analytics

- Aka Supercharged Manual Design
- Via Active Learning / Fast Combination Sweep
- And Predictive Analytics / Knowledge Extraction
- Efficient & optimal (like opt)
- Maintains user control & insight (like manual)